

ISC Class 12 Applied Mathematics Specimen Paper 2

Written by the Future Topper subject team on the CISCE examination pattern. Not an official CISCE paper.
Found a mistake? Write to contactfuturetopper@gmail.com

Maximum Marks: 80 | Time Allowed: Three hours | Reading Time: 15 minutes

All 20 questions are compulsory. Internal choice is offered in three questions in each of Sections B, C and D.

General Instructions

Write your answers on the separate answer paper provided. The first 15 minutes are for reading the paper, and you may not write during that time. The paper has four sections, A, B, C and D, and 20 questions, all of which are compulsory. Section A has one question of twenty parts carrying 1 mark each; Section B has seven questions of 2 marks each; Section C has seven questions of 3 marks each; Section D has five questions of 5 marks each. Internal choices are provided in three questions in each of Sections B, C and D: where a question is followed by OR, attempt only one of the two. For the multiple choice parts of Section A write only the option you choose. Marks for each question, or part of a question, are shown in brackets. All working, including rough work, must be shown on the same sheet as the answer, and omission of essential working will lose marks. Mathematical tables and graph paper are provided. Where a power, an exponential value or a value from the normal table is needed, it is stated in the question and should be used as given. No diagrams, graphs or tables are printed in this paper: data are given in words, and where a graph is needed you draw it yourself on the graph paper. This paper carries 80 of the 100 marks for the subject, the remaining 20 being for the two projects that make up the Project Work.

SECTION A (20 Marks)

Attempt all parts of Question 1.

Question 1 [20]

Each part carries 1 mark. For parts (i) to (xvii) write only the option you choose. For parts (xviii) to (xx) give the answer in one or two lines.

- (i) The value of $\cos^{-1}(-1/2) + \sin^{-1}(1/2)$, using principal values, is:
(a) $\pi/2$ (b) $5\pi/6$ (c) π (d) $2\pi/3$
- (ii) A is the matrix whose first row is 2, 3 and whose second row is 1, 4, and I is the identity matrix of order 2. Then $A^2 - 6A$ equals:
(a) $5I$ (b) $6I$ (c) $-5I$ (d) the zero matrix
- (iii) A and B are square matrices of the same order. The transpose of AB is:
(a) $A'B'$ (b) AB (c) BA (d) $B'A'$
- (iv) The area of the triangle with vertices (0, 0), (4, 0) and (0, k), where $k > 0$, is 6 square units. Then k is:
(a) 3 (b) 2 (c) 6 (d) 12
- (v) If $y = \log(x^2 + 1)$, then the value of dy/dx at $x = 1$ is:
(a) $1/2$ (b) 2 (c) 1 (d) $\log 2$
- (vi) The radius of a circular oil patch on a lake is increasing at 2 metres per minute. When the radius is 10 metres, the area of the patch is increasing at:
(a) 20π square metres per minute (b) 4π square metres per minute
(c) 100π square metres per minute (d) 40π square metres per minute
- (vii) The value of the integral of $(3x^2 + 1)$ with respect to x from 0 to 2 is:
(a) 10 (b) 8 (c) 12 (d) 14
- (viii) Assertion (A): The revenue function $R(x) = 40x - x^2$ has its greatest value at $x = 20$. Reason (R): At $x = 20$ the marginal revenue is zero and the derivative of the marginal revenue is negative.

- (a) A and R are both true, and R explains A. (b) A and R are both true, but R does not explain A.
 (c) A is true but R is false. (d) A is false but R is true.
- (ix) For events A and B, $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \text{ or } B) = 0.7$. Then $P(A \text{ given } B)$ is:
 (a) 0.2 (b) 0.5 (c) 0.4 (d) 0.8
- (x) Statement I: For a Poisson distribution, the mean is equal to the variance. Statement II: For a binomial distribution, the mean is always less than the variance.
 (a) Both the statements are true. (b) Both the statements are false.
 (c) Statement I is true and Statement II is false. (d) Statement I is false and Statement II is true.
- (xi) A random variable X is normally distributed with mean 60 and standard deviation 5. The standard normal variate corresponding to $X = 70$ is:
 (a) -2 (b) 0.5 (c) 10 (d) 2
- (xii) Statement I: The feasible region of every linear programming problem is bounded. Statement II: If the feasible region is bounded, the objective function has both a maximum and a minimum value, each occurring at a corner point.
 (a) Both the statements are true. (b) Both the statements are false.
 (c) Statement I is true and Statement II is false. (d) Statement I is false and Statement II is true.
- (xiii) A loan of Rs 60,000 is taken at a flat rate of 10 per cent per annum for 2 years and is repaid in 24 equal monthly instalments. Each instalment is:
 (a) Rs 2,500 (b) Rs 3,000 (c) Rs 2,750 (d) Rs 3,250
- (xiv) Which of the following correctly describes a sinking fund?
 (a) Regular deposits to meet a known future liability (b) A loan taken to buy a fixed asset
 (c) Equal payments that continue for ever (d) The fall in an asset's value through use
- (xv) Assertion (A): Under the straight line method, the book value of a machine falls by the same amount every year. Reason (R): Under the straight line method, the depreciation for each year is a fixed percentage of the book value at the start of that year.
 (a) A and R are both true, and R explains A. (b) A and R are both true, but R does not explain A.
 (c) A is true but R is false. (d) A is false but R is true.
- (xvi) The price relatives of two commodities are 120 and 150, and their weights are 3 and 2 respectively. The weighted average of price relatives is:
 (a) 135 (b) 130 (c) 132 (d) 138
- (xvii) The present value of a perpetuity of Rs 1,500 paid at the end of every quarter, at 12 per cent per annum compounded quarterly, is:
 (a) Rs 12,500 (b) Rs 50,000 (c) Rs 1,50,000 (d) Rs 5,000
- (xviii) Write the integrating factor of the differential equation $dy/dx - y/x = x^2$, where $x > 0$.
- (xix) The corner points of the feasible region of a linear programming problem are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Find the minimum value of $Z = 4x + 6y$.
- (xx) A binomial distribution has mean 4 and variance 3. Find the number of trials n.

SECTION B (14 Marks)

Attempt all questions. Questions 2 to 8 carry 2 marks each. Internal choice is given in three questions.

Question 2 [2]

A relation R on the set Z of integers is defined by a R b if and only if $a - b$ is divisible by 4. Show that R is an equivalence relation. [2]

Question 3 [2]

Using Cramer's rule, solve the equations $3x + 2y = 12$ and $5x - 3y = 1$. [2]

OR

Question 3 [2]

A is the matrix whose first row is 4, 1 and whose second row is -2, 1. Show that $A^2 - 5A + 6I = O$, where I is the identity matrix and O the zero matrix of order 2, and hence find A^{-1} . [2]

Question 4 [2]

If $x^2 + xy + y^2 = 7$, find dy/dx at the point (1, 2). [2]

Question 5 [2]

Differentiate $(\sin x)^x$ with respect to x, for $0 < x < \pi$. [2]

Question 6 [2]

In a class of 40 students in Kolkata, 24 play cricket, 18 play football and 10 play both. A student is chosen at random and is found to play football. Find the probability that the student also plays cricket, and state, with a reason, whether the events 'plays cricket' and 'plays football' are independent. [2]

OR

Question 6 [2]

A random variable X takes the values 1, 2, 3 and 4 with $P(X = x) = kx$. Find k and $P(X \geq 3)$. [2]

Question 7 [2]

The annual revenue of a start-up in Bengaluru grew from Rs 40 lakh in 2021 to Rs 60 lakh in 2025. Find its compound annual growth rate (CAGR) over these four years, correct to two decimal places of a per cent. Use $(1.5)^{1/4} = 1.1067$. [2]

Question 8 [2]

A household in Mysuru buys 10 kg of rice, 5 litres of cooking oil and 20 litres of milk in a month. In 2020 the prices were Rs 40 per kg of rice, Rs 120 per litre of oil and Rs 50 per litre of milk; in 2025 they were Rs 50, Rs 150 and Rs 55 respectively. Using these base year quantities as weights, find the weighted aggregative price index for 2025 with 2020 as base. [2]

OR

Question 8 [2]

For a group of factory workers in Jamshedpur, the group indices for 2025 (base 2020) and the weights are: food, index 120, weight 50; house rent, index 150, weight 20; clothing, index 110, weight 10; miscellaneous, index 130, weight 20. Find the cost of living index for 2025, and the monthly income a worker who earned Rs 30,000 in 2020 needs in 2025 to keep the same standard of living. [2]

SECTION C (21 Marks)

Attempt all questions. Questions 9 to 15 carry 3 marks each. Internal choice is given in three questions.

Question 9 [3]

Find the value of $\tan[\sin^{-1}(5/13) + \cos^{-1}(4/5)]$. [3]

Question 10 [3]

A farmer in Punjab has 600 metres of fencing to enclose a rectangular field that lies along a straight canal. No fencing is needed along the canal. Find the dimensions of the field of greatest area and that greatest area. [3]

OR

Question 10 [3]

Find the intervals on which the function $f(x) = 2x^3 - 3x^2 - 36x + 7$ is increasing and the interval on which it is decreasing. [3]

Question 11 [3]

Evaluate the integral of $(x^2 + 1)/((x - 1)(x - 2))$ with respect to x . [3]

Question 12 [3]

The number of calls received per minute at a customer helpline in Gurugram follows a Poisson distribution with mean 2. Use $e^{-2} = 0.1353$.

- (a) Find the probability that at least one call is received in a given minute. [1]
- (b) Find the probability that at most two calls are received in a given minute. [2]

OR

Question 12 [3]

A quiz has 5 multiple choice questions, each with 4 options of which exactly one is correct. Rohan guesses every answer at random.

- (a) Find the probability that he gets at least 4 answers correct. [2]
- (b) Find the expected number of correct answers. [1]

Question 13 [3]

A hostel warden in Shimla mixes two foods, P and Q. One kilogram of P contains 2 units of vitamin A and 1 unit of vitamin B; one kilogram of Q contains 1 unit of vitamin A and 2 units of vitamin B. The mixture must contain at least 8 units of vitamin A and at least 10 units of vitamin B. P costs Rs 50 per kg and Q costs Rs 70 per kg. Formulate this as a linear programming problem and solve it graphically, drawing the feasible region yourself, to find the least cost. [3]

Question 14 [3]

Mr Nair bought a shop in Indore for Rs 40,00,000. During the year he received rent of Rs 1,80,000 after meeting all expenses, and at the end of the year he sold the shop for Rs 42,60,000. Ignore taxes and fees on the sale.

- (a) Find his total gain for the year. [1]
- (b) Find his rate of return. [1]
- (c) Explain why the rate found in (b) is called a nominal rate of return. [1]

OR

Question 14 [3]

A printing press in Kochi buys a machine for Rs 3,50,000. Its useful life is 6 years and its scrap value at the end of that time is Rs 50,000. Depreciation is charged by the straight line method.

- (a) Find the annual depreciation and the rate of depreciation as a percentage of the cost. [2]
- (b) Find the book value of the machine at the end of 4 years. [1]

Question 15 [3]

The sales of a garment exporter in Tiruppur, in crore rupees, for the years 2017 to 2024 were 30, 34, 38, 36, 40, 44, 42 and 46 respectively. Calculate the four-yearly centred moving averages, and plot the original figures and the moving averages on the same graph paper. [3]

SECTION D (25 Marks)

Attempt all questions. Questions 16 to 20 carry 5 marks each. Internal choice is given in three questions.

Question 16 [5]

A cafe in Shillong sells tea at Rs x a cup, sandwiches at Rs y each and slices of cake at Rs z each. One table pays Rs 150 for 1 tea, 2 sandwiches and 1 cake; a second pays Rs 200 for 3 teas, 1 sandwich and 2 cakes; a third pays Rs 210 for 1 tea, 1 sandwich and 3 cakes. A is the matrix with rows 1, 2, 1; 3, 1, 2; 1, 1, 3 and B is the matrix with rows 1, -5, 3; -7, 2, 1; 2, 1, -5.

- Find the product AB and hence write down A^{-1} . [2]
- Write the information as a system of equations and use A^{-1} to find x , y and z . [2]
- Find the bill for 3 teas, 2 sandwiches and 2 slices of cake. [1]

Question 17 [5]

Consider the differential equation $dy/dx = e^{x-y} + x^2e^{-y}$.

- Separate the variables. [1]
- Find the general solution. [2]
- Find the particular solution given that $y = 0$ when $x = 0$, and write y explicitly in terms of x . [2]

OR

Question 17 [5]

Consider the differential equation $(1 + x^2)(dy/dx) + 2xy = 4x^2$.

- Write the equation in the form $dy/dx + Py = Q$ and find the integrating factor. [2]
- Find the general solution. [2]
- Find the particular solution given that $y = 0$ when $x = 0$. [1]

Question 18 [5]

The total cost in rupees of making x units of a hand-woven shawl in a week is $C(x) = 0.5x^2 + 30x + 800$.

- Write down the marginal cost and the average cost functions. [1]
- Find the weekly output at which the average cost is least, and that least average cost, using the second derivative test. [3]
- Verify that at this output the marginal cost equals the average cost. [1]

OR

Question 18 [5]

A company's marginal revenue function for a brand of ceiling fan is $MR = 120 - 6x$, where x is the number of fans sold (in hundreds) and revenue is in thousands of rupees.

- Find the total revenue function and the demand function. [2]
- Find the sales at which the total revenue is greatest, and the greatest revenue. [2]
- Find the price at this level of sales. [1]

Question 19 [5]

The working life of LED bulbs made by a factory in Vadodara is normally distributed with mean 1200 hours and standard deviation 100 hours. A shop receives a batch of 2000 such bulbs. Use $P(0 < Z < 1) = 0.3413$, $P(0 < Z < 1.5) = 0.4332$ and $P(0 < Z < 2) = 0.4772$, where Z is the standard normal variate.

- Find how many bulbs in the batch can be expected to last more than 1350 hours. [2]
- Find how many can be expected to last between 1100 and 1400 hours. [2]
- Find how many can be expected to last less than 1000 hours. [1]

OR

Question 19 [5]

In a town, 1 per cent of the people have a certain disease. A screening test gives a positive result for 95 per cent of the people who have the disease, and also gives a positive result for 4 per cent of the people who do not have it. A person chosen at random is tested.

- (a) Define suitable events and write down the probabilities given. [1]
- (b) Find the probability that the test result is positive. [2]
- (c) The test result is positive. Find the probability that the person has the disease, and comment on the result. [2]

Question 20 [5]

A family in Varanasi wishes to set up an endowment that will pay a scholarship of Rs 48,000 every year, for ever, to the best Class 12 student of a local school. Money can be invested at 8 per cent per annum compounded annually. Use $(1.08)^{-3} = 0.7938$.

- (a) Find the sum to be invested now if the first scholarship is paid at the end of one year. [1]
- (b) The school asks for the first scholarship to be paid at the end of 4 years instead, the payments then continuing every year for ever. Find the sum to be invested now. [2]
- (c) If the interest rate available were 6 per cent per annum instead, find the extra sum needed for the arrangement in (a). [1]
- (d) Explain why a finite sum can pay a scholarship for ever. [1]

ANSWER KEY AND MARKING SCHEME

A full model answer is given for every part. Marks shown in italics are the step marks an examiner would award, and they add up to the marks for the part. Method marks are given for a correct method even when an arithmetical slip spoils the final figure, but the final answer mark needs the correct value with its unit (rupees, units, square metres and so on). Accept any correct alternative method that reaches the same answer. In financial questions accept answers that differ only in the last rupee or paise through rounding, and accept values worked with the exact powers instead of the rounded ones printed in the question. For a question with an OR alternative, mark only the alternative the candidate has attempted; if both are attempted, mark both and keep the better mark. Every numerical result in this key has been recomputed by the Future Topper subject team in Python.

Question 1 [20]

- (i) (b) $5\pi/6$. $\cos^{-1}(-1/2) = \pi - \pi/3 = 2\pi/3$, which lies in $[0, \pi]$, and $\sin^{-1}(1/2) = \pi/6$, so the sum is $5\pi/6$.
- (ii) (c) $-5I$. A^2 has rows 7, 18 and 6, 19; $6A$ has rows 12, 18 and 6, 24; the difference has rows -5 , 0 and 0, -5 , which is $-5I$.
- (iii) (d) $B'A'$, by the reversal law for transposes.
- (iv) (a) 3. The area is $(1/2)|0(0 - k) + 4(k - 0) + 0(0 - 0)| = 2k$, and $2k = 6$ gives $k = 3$.
- (v) (c) 1. $dy/dx = 2x/(x^2 + 1)$, which is $2/2 = 1$ at $x = 1$.
- (vi) (d) 40π square metres per minute. $A = \pi r^2$, so $dA/dt = 2\pi r(dr/dt) = 2\pi \times 10 \times 2 = 40\pi$.
- (vii) (a) 10. $[x^3 + x]$ from 0 to 2 = $8 + 2 = 10$.
- (viii) (a) Both are true and the Reason explains the Assertion. $R'(x) = 40 - 2x = 0$ at $x = 20$ and $R''(x) = -2 < 0$.
- (ix) (b) 0.5. $P(A \text{ and } B) = 0.5 + 0.4 - 0.7 = 0.2$, so $P(A \text{ given } B) = 0.2/0.4 = 0.5$.
- (x) (c) Statement I is true and Statement II is false. For the binomial distribution the variance npq is less than the mean np , since $q < 1$.
- (xi) (d) 2. $Z = (X - \text{mean})/\text{standard deviation} = (70 - 60)/5 = 2$.
- (xii) (d) Statement I is false and Statement II is true. Feasible regions may be unbounded (for example in diet problems), while Statement II is the corner point theorem.
- (xiii) (b) Rs 3,000. Interest = $60,000 \times 0.10 \times 2 = 12,000$, and $(60,000 + 12,000)/24 = 3,000$.
- (xiv) (a) Regular deposits to meet a known future liability: a sinking fund is built up by such deposits. Option (c) describes a perpetuity and option (d) describes depreciation.
- (xv) (c) The Assertion is true but the Reason is false. A fixed percentage of the opening book value describes the written down value method; the straight line method charges a fixed amount each year.
- (xvi) (c) 132. $(120 \times 3 + 150 \times 2)/(3 + 2) = 660/5 = 132$.
- (xvii) (b) Rs 50,000. The rate per quarter is 3 per cent, so the present value is $1,500/0.03 = 50,000$.
- (xviii) The integrating factor is e to the power of the integral of $(-1/x) dx = e^{-\log x} = 1/x$.
- (xix) The values of Z are 12, 12, 24, 72 and 30, so the minimum is 12. It occurs at both (0, 2) and (3, 0), and so at every point of the segment joining them.
- (xx) $np = 4$ and $npq = 3$, so $q = 3/4$ and $p = 1/4$, giving $n = 4/(1/4) = 16$.

1 mark for each correct part. No mark is deducted for a wrong option. In parts (xviii) to (xx) the mark is given for the correct value; a one-line reason is welcome but not required.

Question 2 [2]

Reflexive: for every integer a , $a - a = 0 = 4 \times 0$, so $a R a$. Symmetric: if $a R b$ then $a - b = 4m$ for some integer m , so $b - a = 4(-m)$ and $b R a$. Transitive: if $a R b$ and $b R c$ then $a - b = 4m$ and $b - c = 4n$, so $a - c = (a - b) + (b - c) = 4(m + n)$ and $a R c$. Being reflexive, symmetric and transitive, R is an equivalence relation.

Reflexive and symmetric shown 1. Transitive shown and conclusion 1.

Question 3 [2]

$D = (3)(-3) - (2)(5) = -9 - 10 = -19$, which is not zero, so there is a unique solution. $D_x = (12)(-3) - (2)(1) = -36 - 2 = -38$ and $D_y = (3)(1) - (12)(5) = 3 - 60 = -57$. Hence $x = D_x/D = 2$ and $y = D_y/D = 3$. Check: $6 + 6 = 12$ and $10 - 9 = 1$.

$D = -19$, $D_x = -38$ and $D_y = -57$ 1. $x = 2$ and $y = 3$ 1.

OR

Question 3 [2]

A^2 has first row $16 - 2, 4 + 1 = 14, 5$ and second row $-8 - 2, -2 + 1 = -10, -1$. $5A$ has rows $20, 5$ and $-10, 5$, and $6I$ has rows $6, 0$ and $0, 6$. So $A^2 - 5A + 6I$ has first row $14 - 20 + 6, 5 - 5 + 0 = 0, 0$ and second row $-10 + 10 + 0, -1 - 5 + 6 = 0, 0$, which is O . Multiplying $A^2 - 5A + 6I = O$ by A^{-1} (A is non-singular, its determinant being 6) gives $A - 5I + 6A^{-1} = O$, so $A^{-1} = (1/6)(5I - A)$, the matrix $(1/6)$ times the matrix with rows $1, -1$ and $2, 4$.

A^2 found and the identity verified 1. $A^{-1} = (1/6)(5I - A)$ with rows $1, -1$ and $2, 4$ 1.

Question 4 [2]

Differentiating implicitly with respect to x : $2x + y + x(dy/dx) + 2y(dy/dx) = 0$, so $dy/dx = -(2x + y)/(x + 2y)$. The point $(1, 2)$ lies on the curve since $1 + 2 + 4 = 7$. At $(1, 2)$, $dy/dx = -(2 + 2)/(1 + 4) = -4/5$.

Implicit differentiation giving $dy/dx = -(2x + y)/(x + 2y)$ 1. Value $-4/5$ 1.

Question 5 [2]

Let $y = (\sin x)^x$. Taking logarithms, $\log y = x \log(\sin x)$. Differentiating, $(1/y)(dy/dx) = \log(\sin x) + x \times (\cos x/\sin x) = \log(\sin x) + x \cot x$. Hence $dy/dx = (\sin x)^x [\log(\sin x) + x \cot x]$.

Logarithms taken and differentiated by the product rule 1. $dy/dx = (\sin x)^x [\log(\sin x) + x \cot x]$ 1.

Question 6 [2]

Let C and F be the events. $P(C \text{ given } F) = P(C \text{ and } F)/P(F) = (10/40)/(18/40) = 10/18 = 5/9$. For independence we need $P(C \text{ and } F) = P(C)P(F)$. Here $P(C)P(F) = (24/40)(18/40) = 0.6 \times 0.45 = 0.27$, while $P(C \text{ and } F) = 10/40 = 0.25$. Since these differ, the events are not independent.

$P(C \text{ given } F) = 5/9$ 1. Comparison 0.25 with 0.27 and conclusion not independent 1.

OR

Question 6 [2]

The probabilities must add to 1: $k + 2k + 3k + 4k = 10k = 1$, so $k = 1/10$. $P(X \geq 3) = P(X = 3) + P(X = 4) = 3/10 + 4/10 = 7/10$.

$k = 1/10$ 1. $P(X \geq 3) = 7/10$ 1.

Question 7 [2]

$\text{CAGR} = (\text{final value}/\text{initial value})^{1/n} - 1$ with $n = 4$. The ratio is $60/40 = 1.5$, so $\text{CAGR} = (1.5)^{1/4} - 1 = 1.1067 - 1 = 0.1067$, that is 10.67 per cent per annum. (Equivalently, $\log(1 + r) = (1/4) \log 1.5 = 0.1761/4 = 0.0440$ using common logarithms, giving $1 + r = 1.1067$.)

Note for the examiner: Accept 10.7 per cent, or a value found from log tables that rounds to it.

Formula with $n = 4$ and ratio 1.5 1. CAGR 10.67 per cent 1.

Question 8 [2]

With base year quantities as weights, the index is $(\text{sum of } p_1 q_0)/(\text{sum of } p_0 q_0) \times 100$. Sum of $p_0 q_0 = 40 \times 10 + 120 \times 5 + 50 \times 20 = 400 + 600 + 1000 = 2000$. Sum of $p_1 q_0 = 50 \times 10 + 150 \times 5 + 55 \times 20 = 500 + 750 + 1100 = 2350$. Index = $(2350/2000) \times 100 = 117.5$.

Both weighted totals, 2000 and 2350 1. Index 117.5 1.

OR

Question 8 [2]

Cost of living index = $(\text{sum of } IW)/(\text{sum of } W) = (120 \times 50 + 150 \times 20 + 110 \times 10 + 130 \times 20)/100 = (6000 + 3000 + 1100 + 2600)/100 = 12700/100 = 127$. The income needed is $30,000 \times 127/100 = \text{Rs } 38,100$ a month.

Cost of living index 127 1. Required income Rs 38,100 1.

Question 9 [3]

Let $A = \sin^{-1}(5/13)$, so $\sin A = 5/13$ with A in the first quadrant; then $\cos A = 12/13$ and $\tan A = 5/12$. Let $B = \cos^{-1}(4/5)$, so $\cos B = 4/5$ with B in the first quadrant; then $\sin B = 3/5$ and $\tan B = 3/4$. Hence $\tan(A + B) = (\tan A + \tan B)/(1 - \tan A \tan B) = (5/12 + 3/4)/(1 - 15/48) = (14/12)/(33/48) = (7/6) \times (48/33) = 56/33$.

$\tan A = 5/12$ 1. $\tan B = 3/4$ 1. Addition formula and value $56/33$ 1.

Question 10 [3]

Let each side perpendicular to the canal be y metres and the side parallel to the canal be x metres. Then $x + 2y = 600$, so $x = 600 - 2y$, and the area is $A = xy = 600y - 2y^2$. $dA/dy = 600 - 4y = 0$ gives $y = 150$, and $d^2A/dy^2 = -4 < 0$, so this is a maximum. Then $x = 600 - 300 = 300$. The field should be 300 metres along the canal and 150 metres wide, with greatest area $300 \times 150 = 45,000$ square metres.

Area expressed as $600y - 2y^2$ 1. $y = 150$ with second derivative test 1. 300 m by 150 m and area 45,000 square metres 1.

OR

Question 10 [3]

$f(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 6(x - 3)(x + 2)$, which is zero at $x = -2$ and $x = 3$. For $x < -2$, say $x = -3$, $f'(-3) = 6(-6)(-1) = 36 > 0$. For $-2 < x < 3$, say $x = 0$, $f'(0) = -36 < 0$. For $x > 3$, say $x = 4$, $f'(4) = 6(1)(6) = 36 > 0$. So f is increasing on the interval from minus infinity to -2 and on the interval from 3 to infinity, and decreasing on the interval $(-2, 3)$.

$f(x) = 6(x - 3)(x + 2)$ 1. Sign of $f'(x)$ tested on all three intervals 1. Intervals stated correctly 1.

Question 11 [3]

The numerator and denominator have the same degree, so divide first: $(x - 1)(x - 2) = x^2 - 3x + 2$, and $x^2 + 1 = (x^2 - 3x + 2) + (3x - 1)$. So the integrand is $1 + (3x - 1)/((x - 1)(x - 2))$. Write $(3x - 1)/((x - 1)(x - 2)) = A/(x - 1) + B/(x - 2)$, so $3x - 1 = A(x - 2) + B(x - 1)$. At $x = 1$, $2 = -A$, so $A = -2$; at $x = 2$, $5 = B$. Hence the integral is $x - 2 \log|x - 1| + 5 \log|x - 2| + c$.

Division giving $1 + (3x - 1)/((x - 1)(x - 2))$ 1. $A = -2$ and $B = 5$ 1. $x - 2 \log|x - 1| + 5 \log|x - 2| + c$ 1.

Question 12 [3]

(a) $P(X \geq 1) = 1 - P(X = 0) = 1 - e^{-2} = 1 - 0.1353 = 0.8647$.

0.8647 1.

(b) $P(X \leq 2) = e^{-2}(1 + 2 + 2^2/2!) = e^{-2}(1 + 2 + 2) = 5 \times 0.1353 = 0.6765$.

Sum of the three Poisson terms set up 1. 0.6765 1.

OR

Question 12 [3]

(a) The number of correct answers X is binomial with $n = 5$, $p = 1/4$ and $q = 3/4$. $P(X \geq 4) = {}^5C_4(1/4)^4(3/4) + (1/4)^5 = 15/1024 + 1/1024 = 16/1024 = 1/64$.

Binomial terms for $X = 4$ and $X = 5$ 1. $1/64$ 1.

(b) Mean $= np = 5 \times 1/4 = 5/4 = 1.25$.

5/4 1.

Question 13 [3]

Let x kg of P and y kg of Q be used. Minimise $Z = 50x + 70y$ subject to $2x + y \geq 8$, $x + 2y \geq 10$, $x \geq 0$ and $y \geq 0$. Draw $2x + y = 8$ through $(4, 0)$ and $(0, 8)$, and $x + 2y = 10$ through $(10, 0)$ and $(0, 5)$. The feasible region lies above both lines in the first quadrant and is unbounded. Its corners are $(0, 8)$, the meeting point of the two lines, which is $(2, 4)$, and $(10, 0)$. $Z(0, 8) = 560$, $Z(2, 4) = 100 + 280 = 380$ and $Z(10, 0) = 500$. Because the region is unbounded, draw $50x + 70y < 380$: this open half-plane has no point in common with the feasible region, so the minimum is Rs 380, using 2 kg of P and 4 kg of Q .

Objective function and constraints formulated 1. Corner points $(0, 8)$, $(2, 4)$ and $(10, 0)$ with values of Z 1. Minimum Rs 380 at $(2, 4)$, with the unbounded region checked 1.

Question 14 [3]

- (a) Capital gain = $42,60,000 - 40,00,000 = \text{Rs } 2,60,000$; net rent = Rs 1,80,000; total gain = Rs 4,40,000.

Total gain Rs 4,40,000 1.

- (b) Rate of return = $\text{total gain/amount invested} \times 100 = 4,40,000/40,00,000 \times 100 = 11 \text{ per cent}$.

11 per cent 1.

- (c) The 11 per cent is the return in rupee terms before any adjustment for inflation (or for taxes), so it does not show the change in his real purchasing power; with inflation the real return is lower.

Correct explanation referring to inflation not being allowed for 1.

OR

Question 14 [3]

- (a) Annual depreciation = $(\text{cost} - \text{scrap value})/\text{life} = (3,50,000 - 50,000)/6 = \text{Rs } 50,000$. Rate = $50,000/3,50,000 \times 100 = 14.29 \text{ per cent}$ (14 $\frac{2}{7}$ per cent) of the cost.

Annual depreciation Rs 50,000 1. Rate 14.29 per cent 1.

- (b) Book value = $3,50,000 - 4 \times 50,000 = \text{Rs } 1,50,000$.

Rs 1,50,000 1.

Question 15 [3]

The four-yearly moving totals are $30 + 34 + 38 + 36 = 138$, then 148, 158, 162 and 172. Dividing by 4, the four-yearly moving averages are 34.5, 37, 39.5, 40.5 and 43; these fall between years (the first between 2018 and 2019). Centring by averaging each successive pair: $(34.5 + 37)/2 = 35.75$ for 2019, $(37 + 39.5)/2 = 38.25$ for 2020, $(39.5 + 40.5)/2 = 40$ for 2021 and $(40.5 + 43)/2 = 41.75$ for 2022. On the graph the centred averages rise steadily, smoothing out the dips of 2020 and 2023 and showing an upward trend.

Four-yearly moving totals and averages 1. Centred averages 35.75, 38.25, 40 and 41.75 placed against 2019 to 2022 1. Both series plotted correctly 1.

Question 16 [5]

- (a) Row 1 of A times the columns of B: $1 - 14 + 2 = -11$, $-5 + 4 + 1 = 0$, $3 + 2 - 5 = 0$. Row 2: $3 - 7 + 4 = 0$, $-15 + 2 + 2 = -11$, $9 + 1 - 10 = 0$. Row 3: $1 - 7 + 6 = 0$, $-5 + 2 + 3 = 0$, $3 + 1 - 15 = -11$. So $AB = -11I$. Hence $A(-B/11) = I$, and $A^{-1} = -(1/11)B$, the matrix $-(1/11)$ times the matrix with rows 1, -5, 3; -7, 2, 1; 2, 1, -5. (B is in fact $\text{adj } A$, and the determinant of A is -11.)

AB computed as $-11I$ 1. $A^{-1} = -(1/11)B$ 1.

- (b) The system is $x + 2y + z = 150$, $3x + y + 2z = 200$ and $x + y + 3z = 210$, that is $AX = C$ with C the column 150, 200, 210. Then $X = A^{-1}C = -(1/11)BC$. BC has entries $150 - 1000 + 630 = -220$, $-1050 + 400 + 210 = -440$ and $300 + 200 - 1050 = -550$. Dividing by -11, $x = 20$, $y = 40$ and $z = 50$: tea costs Rs 20 a cup, a sandwich Rs 40 and a slice of cake Rs 50. (Check in the second bill: $60 + 40 + 100 = 200$.)

System written as $AX = C$ and $X = -(1/11)BC$ 1. $x = 20$, $y = 40$, $z = 50$ 1.

- (c) Bill = $3 \times 20 + 2 \times 40 + 2 \times 50 = 60 + 80 + 100 = \text{Rs } 240$.

Rs 240 1.

Question 17 [5]

- (a) $dy/dx = e^{-y}(e^x + x^2)$, so $e^y dy = (e^x + x^2) dx$.

Variables separated 1.

- (b) Integrating both sides, $e^y = e^x + x^3/3 + c$, where c is an arbitrary constant.

Both sides integrated 1. $e^y = e^x + x^3/3 + c$ 1.

- (c) At $x = 0$, $y = 0$: $1 = 1 + 0 + c$, so $c = 0$. Hence $e^y = e^x + x^3/3$, and $y = \log(e^x + x^3/3)$.

$c = 0$ 1. $y = \log(e^x + x^3/3)$ 1.

OR

Question 17 [5]

- (a) Dividing by $1 + x^2$: $dy/dx + [2x/(1 + x^2)]y = 4x^2/(1 + x^2)$. The integral of $2x/(1 + x^2) dx$ is $\log(1 + x^2)$, so the integrating factor is $1 + x^2$.

P and Q identified 1. Integrating factor $1 + x^2$ 1.

- (b) Multiplying by the integrating factor, $d/dx[(1 + x^2)y] = 4x^2$. Integrating, $(1 + x^2)y = 4x^3/3 + c$.

$d/dx[(1 + x^2)y] = 4x^2$ 1. $(1 + x^2)y = 4x^3/3 + c$ 1.

- (c) At $x = 0$: $0 = 0 + c$, so $c = 0$ and $y = 4x^3/(3(1 + x^2))$.

$y = 4x^3/(3(1 + x^2))$ 1.

Question 18 [5]

- (a) Marginal cost $MC = C'(x) = x + 30$. Average cost $AC = C(x)/x = 0.5x + 30 + 800/x$.

$MC = x + 30$ and $AC = 0.5x + 30 + 800/x$ 1.

- (b) $d(AC)/dx = 0.5 - 800/x^2$, which is zero when $x^2 = 1600$, that is $x = 40$ (x being positive). $d^2(AC)/dx^2 = 1600/x^3$, which is positive at $x = 40$, so this is a minimum. The least average cost is $0.5 \times 40 + 30 + 800/40 = 20 + 30 + 20 = \text{Rs } 70$ per shawl.

$d(AC)/dx$ set to zero 1. $x = 40$ with second derivative positive 1. Least average cost Rs 70 1.

- (c) MC at $x = 40$ is $40 + 30 = 70$, which equals the least average cost of Rs 70.

$MC(40) = 70 = AC(40)$ 1.

OR

Question 18 [5]

- (a) $R(x)$ is the integral of $(120 - 6x) dx = 120x - 3x^2 + k$. Revenue is zero when nothing is sold, so $k = 0$ and $R(x) = 120x - 3x^2$. Since $R = px$, the demand function is $p = R/x = 120 - 3x$.

$R(x) = 120x - 3x^2$ with $k = 0$ justified 1. Demand function $p = 120 - 3x$ 1.

- (b) $MR = 120 - 6x = 0$ when $x = 20$, and $d(MR)/dx = -6 < 0$, so revenue is greatest at $x = 20$ (that is 2,000 fans). $R(20) = 2400 - 1200 = 1200$, that is Rs 12,00,000.

$x = 20$ with the second derivative test 1. Greatest revenue 1200 thousand rupees 1.

- (c) $p = 120 - 3 \times 20 = 60$, that is Rs 60 thousand per hundred fans, or Rs 600 per fan.

Price 60 (Rs 600 per fan) 1.

Question 19 [5]

- (a) $Z = (1350 - 1200)/100 = 1.5$. $P(Z > 1.5) = 0.5 - 0.4332 = 0.0668$. Expected number = $0.0668 \times 2000 = 133.6$, that is about 134 bulbs.

$Z = 1.5$ and probability 0.0668 1. About 134 bulbs 1.

- (b) For 1100, $Z = -1$; for 1400, $Z = 2$. $P(-1 < Z < 2) = 0.3413 + 0.4772 = 0.8185$. Expected number = $0.8185 \times 2000 = 1637$ bulbs.

Z values -1 and 2 , probability 0.8185 1. 1637 bulbs 1.

- (c) $Z = (1000 - 1200)/100 = -2$. $P(Z < -2) = 0.5 - 0.4772 = 0.0228$, so the expected number is $0.0228 \times 2000 = 45.6$, that is about 46 bulbs.

About 46 bulbs 1.

OR

Question 19 [5]

- (a) Let D be the event that the person has the disease and T the event that the test is positive. $P(D) = 0.01$, $P(\text{not } D) = 0.99$, $P(T \text{ given } D) = 0.95$ and $P(T \text{ given not } D) = 0.04$.

Events defined with all four probabilities 1.

- (b) By the theorem of total probability, $P(T) = P(D)P(T \text{ given } D) + P(\text{not } D)P(T \text{ given not } D) = 0.01 \times 0.95 + 0.99 \times 0.04 = 0.0095 + 0.0396 = 0.0491$.

Total probability set up 1. $P(T) = 0.0491$ 1.

- (c) By Bayes' theorem, $P(D \text{ given } T) = 0.0095/0.0491 = 95/491$, which is about 0.193. So fewer than one in five people who test positive actually have the disease; because the disease is rare, most positive results are false positives, and a positive result should be confirmed by a further test.

Bayes' theorem giving 95/491 (about 0.193) 1. Sensible comment on the low value 1.

Question 20 [5]

- (a) This is an ordinary perpetuity: present value = $R/i = 48,000/0.08 = \text{Rs } 6,00,000$.

Rs 6,00,000 1.

- (b) This is a perpetuity deferred by 3 years. At the end of the third year, one year before the first payment, the perpetuity is worth $48,000/0.08 = \text{Rs } 6,00,000$. Discounting this back 3 years, the sum needed now is $6,00,000 \times (1.08)^{-3} = 6,00,000 \times 0.7938 = \text{Rs } 4,76,280$.

Value Rs 6,00,000 at the end of year 3 1. Present value Rs 4,76,280 1.

- (c) At 6 per cent the present value is $48,000/0.06 = \text{Rs } 8,00,000$, so an extra $8,00,000 - 6,00,000 = \text{Rs } 2,00,000$ is needed.

Extra Rs 2,00,000 1.

- (d) The payments are made out of the interest alone: Rs 6,00,000 at 8 per cent earns exactly Rs 48,000 a year, so the capital is never used up. Equivalently, the present values of later payments become smaller and smaller, and their sum (a geometric series) is the finite value R/i .

Valid explanation (interest only is paid out, or the geometric series converges) 1.

Note for the examiner: With the exact power $(1.08)^{-3} = 0.793832$ the answer to (b) is Rs 4,76,299; accept either.