

ISC Class 12 Applied Mathematics Specimen Paper 1

Written by the Future Topper subject team on the CISCE examination pattern. Not an official CISCE paper.
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Maximum Marks: 80 | Time Allowed: Three hours | Reading Time: 15 minutes

All 20 questions are compulsory. Internal choice is offered in three questions in each of Sections B, C and D.

General Instructions

Write your answers on the separate answer paper provided. The first 15 minutes are for reading the paper, and you may not write during that time. The paper has four sections, A, B, C and D, and 20 questions, all of which are compulsory. Section A has one question of twenty parts carrying 1 mark each; Section B has seven questions of 2 marks each; Section C has seven questions of 3 marks each; Section D has five questions of 5 marks each. Internal choices are provided in three questions in each of Sections B, C and D: where a question is followed by OR, attempt only one of the two. For the multiple choice parts of Section A write only the option you choose. Marks for each question, or part of a question, are shown in brackets. All working, including rough work, must be shown on the same sheet as the answer, and omission of essential working will lose marks. Mathematical tables and graph paper are provided. Where a power, an exponential value or a value from the normal table is needed, it is stated in the question and should be used as given. No diagrams, graphs or tables are printed in this paper: data are given in words, and where a graph is needed you draw it yourself on the graph paper. This paper carries 80 of the 100 marks for the subject, the remaining 20 being for the two projects that make up the Project Work.

SECTION A (20 Marks)

Attempt all parts of Question 1.

Question 1 [20]

Each part carries 1 mark. For parts (i) to (xvii) write only the option you choose. For parts (xviii) to (xx) give the answer in one or two lines.

- (i) The relation R on the set $\{1, 2, 3\}$ is given by $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$. Then R is:
(a) reflexive and symmetric but not transitive (b) an equivalence relation
(c) symmetric and transitive but not reflexive (d) reflexive and transitive but not symmetric
- (ii) A is a square matrix of order 3 and the determinant of A is 5. The determinant of $\text{adj } A$ is:
(a) 5 (b) 15 (c) 25 (d) 125
- (iii) A is any square matrix and A' is its transpose. The matrix $A - A'$ is always:
(a) skew symmetric (b) symmetric
(c) a diagonal matrix (d) the zero matrix
- (iv) If $y = x^2 e^x$, then dy/dx is:
(a) $2xe^x$ (b) $x^2 e^x$
(c) $xe^x(x + 2)$ (d) $2xe^x + e^x$
- (v) The integral of $1/(x \log x)$ with respect to x , for $x > 1$, is:
(a) $\log x + c$ (b) $1/(\log x) + c$
(c) $(\log x)^2/2 + c$ (d) $\log|\log x| + c$
- (vi) For a binomial distribution with $n = 10$ and $p = 0.4$, the variance is:
(a) 4 (b) 2.4 (c) 6 (d) 1.6
- (vii) The corner points of the bounded feasible region of a linear programming problem are $(0, 0)$, $(4, 0)$, $(3, 2)$ and $(0, 3)$. The maximum value of $Z = 2x + 3y$ is:
(a) 8 (b) 9 (c) 12 (d) 13

- (viii) Assertion (A): The present value of a perpetuity of Rs 600 paid at the end of every year, at 8 per cent per annum, is Rs 7,500. Reason (R): The present value of a perpetuity of R rupees paid at the end of each period, at a rate i per period, is R/i .
- (a) A and R are both true, and R explains A. (b) A and R are both true, but R does not explain A.
(c) A is true but R is false. (d) A is false but R is true.
- (ix) The total cost in rupees of producing x units is $C(x) = 2x^2 + 5x + 100$. The marginal cost when 10 units are produced is:
- (a) Rs 45 (b) Rs 405 (c) Rs 25 (d) Rs 350
- (x) Statement I: If A and B are independent events, then A and the complement of B are also independent. Statement II: For any two events A and B with $P(B)$ not zero, $P(A \text{ given } B) = P(A \text{ and } B)/P(B)$.
- (a) Both the statements are true. (b) Both the statements are false.
(c) Statement I is true and Statement II is false. (d) Statement I is false and Statement II is true.
- (xi) A machine bought for Rs 80,000 is expected to have a scrap value of Rs 8,000 after 8 years. The annual depreciation by the straight line method is:
- (a) Rs 10,000 (b) Rs 9,000 (c) Rs 8,000 (d) Rs 11,000
- (xii) Statement I: The function $f(x) = x^2 - 4x$ is increasing on the interval $(0, 4)$. Statement II: A function is decreasing on an interval if its derivative is negative at every point of that interval.
- (a) Both the statements are true. (b) Both the statements are false.
(c) Statement I is true and Statement II is false. (d) Statement I is false and Statement II is true.
- (xiii) The system of equations $2x + 3y = 5$ and $4x + 6y = 12$ has:
- (a) a unique solution (b) infinitely many solutions
(c) no solution (d) exactly two solutions
- (xiv) A random variable X follows the Poisson distribution with mean 2. Then $P(X = 0)$ is:
- (a) e^{-2} (b) $2e^{-2}$
(c) $1 - e^{-2}$ (d) e^2
- (xv) Assertion (A): For the same loan amount, rate and term, the flat rate method charges less total interest than the reducing balance method. Reason (R): In the flat rate method, interest is charged on the full original principal for the whole term.
- (a) A and R are both true, and R explains A. (b) A and R are both true, but R does not explain A.
(c) A is true but R is false. (d) A is false but R is true.
- (xvi) The degree of the differential equation $d^2y/dx^2 + 3(dy/dx)^2 = x^2 \log(d^2y/dx^2)$ is:
- (a) 1 (b) 2 (c) 3 (d) not defined
- (xvii) A random variable X takes the values 0, 1 and 2 with probabilities k , $2k$ and $3k$ respectively. The mean of X is:
- (a) 1 (b) $4/3$ (c) $5/3$ (d) 2
- (xviii) The price of rice in a market in Patna was Rs 40 per kg in 2020 and Rs 52 per kg in 2025. Find the price relative for 2025 taking 2020 as the base year.
- (xix) A tailor has at most 40 metres of cloth. A shirt needs 2 metres and a kurta needs 3 metres. Taking x shirts and y kurtas, write the constraint on cloth.
- (xx) Neha bought a share for Rs 250, received a dividend of Rs 10 during the year and sold the share at the end of the year for Rs 270. Find her rate of return for the year.

SECTION B (14 Marks)

Attempt all questions. Questions 2 to 8 carry 2 marks each. Internal choice is given in three questions.

Question 2 [2]

The functions f and g from the set of real numbers to itself are given by $f(x) = 3x - 2$ and $g(x) = x^2 + 1$.

Find $(g \circ f)(x)$ and the value of $(f \circ g)(2)$. [2]

Question 3 [2]

A farmer near Nashik marks out a triangular plot whose corners, measured in metres from a fixed post, are at the points $(1, 2)$, $(4, 6)$ and $(7, 3)$. Using determinants, find the area of the plot. [2]

OR

Question 3 [2]

Using determinants, find the value of k for which the points $(2, 3)$, $(5, k)$ and $(8, 9)$ lie on a straight line. [2]

Question 4 [2]

If $x = 2t + 3$ and $y = t^3 - 1$, find dy/dx and d^2y/dx^2 when $t = 2$. [2]

Question 5 [2]

At an electric bulb factory in Noida, 10 per cent of the bulbs produced are defective. An inspector picks 4 bulbs at random, the picks being independent. Find the probability that at most one of the 4 bulbs is defective. [2]

OR

Question 5 [2]

A fair die is thrown three times. Find the probability of getting a six on exactly two of the throws. [2]

Question 6 [2]

A charitable trust in Chennai wishes to award a scholarship of Rs 30,000 every year for ever, the money being invested at 6 per cent per annum compounded annually. Find the sum to be invested now if:

- (a) the first scholarship is paid at the end of the first year; [1]
- (b) the first scholarship is paid immediately. [1]

Question 7 [2]

The retail prices in rupees of four items in a market in Bhopal were as follows. Wheat: Rs 30 per kg in 2020 and Rs 36 per kg in 2025. Rice: Rs 50 per kg in 2020 and Rs 60 per kg in 2025. Pulses: Rs 80 per kg in 2020 and Rs 105 per kg in 2025. Sugar: Rs 40 per kg in 2020 and Rs 49 per kg in 2025. Calculate the price index for 2025 with 2020 as base by the simple aggregative method. [2]

Question 8 [2]

Evaluate the definite integral of $|x - 2|$ with respect to x from 0 to 4. [2]

OR

Question 8 [2]

Using the property that the integral of $f(x)$ from 0 to a equals the integral of $f(a - x)$ from 0 to a , evaluate the integral of $x(1 - x)^5$ with respect to x from 0 to 1. [2]

SECTION C (21 Marks)

Attempt all questions. Questions 9 to 15 carry 3 marks each. Internal choice is given in three questions.

Question 9 [3]

Solve for x : $\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$. [3]

OR

Question 9 [3]

Prove that $2 \tan^{-1}(1/2) - \tan^{-1}(1/7) = \pi/4$. [3]

Question 10 [3]

A dairy cooperative in Anand finds that when it sells x litres of flavoured milk a day, the price it can charge is $p = 80 - 0.02x$ rupees per litre. The daily cost of producing x litres is $C(x) = 20x + 5000$ rupees. Find the daily sales that give the greatest profit, the price per litre at that level and the greatest daily profit. [3]

Question 11 [3]

Evaluate the integral of $(5x + 4)/((x - 1)(x + 2))$ with respect to x . [3]

OR

Question 11 [3]

Evaluate the integral of xe^{2x} with respect to x from 0 to 1. [3]

Question 12 [3]

A box in a mobile repair shop contains 6 chargers, of which 2 are faulty. Two chargers are drawn at random without replacement. Let X be the number of faulty chargers drawn. Find the probability distribution of X , and hence its mean and variance. [3]

Question 13 [3]

Arjun takes a car loan of Rs 6,00,000 from a bank at 12 per cent per annum, interest being charged monthly on the reducing balance (that is, 1 per cent per month). The loan is to be repaid in 36 equal monthly instalments, the first at the end of the first month. Use $(1.01)^{36} = 1.4308$.

- Find the equated monthly instalment (EMI), to the nearest rupee. [2]
- Find the total interest Arjun pays over the term of the loan. [1]

OR

Question 13 [3]

Priya needs a loan of Rs 1,20,000 for a motorcycle, to be repaid in 24 equal monthly instalments. Lender P charges a flat rate of 9 per cent per annum. Lender Q charges 15 per cent per annum on the reducing balance, that is 1.25 per cent per month. Use $(1.0125)^{24} = 1.3474$.

- Find the total interest and the monthly instalment with Lender P. [1]
- Find the EMI with Lender Q. [1]
- Find the total interest with Lender Q, and state which lender is cheaper and by how much, even though its quoted rate is higher. [1]

Question 14 [3]

Solve the following linear programming problem graphically, drawing the feasible region yourself on graph paper. Maximise $Z = 5x + 4y$ subject to $x + y \leq 8$, $x + 2y \leq 12$, $x \leq 6$, $x \geq 0$ and $y \geq 0$. [3]

Question 15 [3]

The production of a sugar mill in Uttar Pradesh, in thousand tonnes, for the years 2018 to 2024 was 42, 48, 45, 51, 57, 54 and 60 respectively. Calculate the three-yearly moving averages, and plot the original figures and the moving averages on the same graph paper. Comment briefly on the trend. [3]

SECTION D (25 Marks)

Attempt all questions. Questions 16 to 20 carry 5 marks each. Internal choice is given in three questions.

Question 16 [5]

At a stationery shop in Lucknow, Riya pays Rs 190 for 1 pen, 1 notebook and 1 geometry box; Kabir pays Rs 450 for 2 pens, 1 notebook and 3 geometry boxes; Meera pays Rs 240 for 1 pen, 2 notebooks and 1 geometry box. Let the prices of a pen, a notebook and a geometry box be Rs x , Rs y and Rs z respectively.

- Write the information as a matrix equation $AX = B$, find the determinant of A and find A^{-1} by the adjoint method. [3]
- Use A^{-1} to find the price of each item. [1]
- The school orders 5 pens and 4 notebooks for a prize. Find the cost of the order. [1]

Question 17 [5]

Consider the differential equation $dy/dx + (2/x)y = 3x$, where $x > 0$.

- Identify the equation as linear, state P and Q , and find the integrating factor. [1]
- Find the general solution. [3]
- Find the particular solution for which $y = 2$ when $x = 1$. [1]

Question 18 [5]

The demand function for a brand of pressure cooker is $p = 50 - x^2$ and the supply function is $p = 2x + 2$, where p is the price in hundreds of rupees and x is the quantity in thousands.

- Find the equilibrium quantity and price. [1]
- Find the consumer surplus at equilibrium. [2]
- Find the producer surplus at equilibrium. [2]

OR

Question 18 [5]

A small factory in Ludhiana makes steel bottles. The marginal cost of producing x bottles a day is $MC = 4x + 20$ rupees, and the fixed cost is Rs 600 a day. Each bottle sells for Rs 100.

- Find the total cost function $C(x)$. [1]
- Find the break-even points. [2]
- Find the daily output that gives the greatest profit, and that profit. [2]

Question 19 [5]

An online shop in Pune sends its parcels through three courier companies, Swift, Rapid and Express. Swift carries 50 per cent of the parcels, Rapid 30 per cent and Express 20 per cent. Of the parcels carried by Swift 2 per cent are delivered late, of those carried by Rapid 5 per cent are late, and of those carried by Express 10 per cent are late.

- Find the probability that a parcel chosen at random is delivered late. [2]
- A parcel is found to have been delivered late. Find the probability that it was carried by Express. [2]
- A parcel is found to have been delivered on time. Find the probability that it was carried by Swift. [1]

OR

Question 19 [5]

The marks of 500 students in a district-level aptitude test are normally distributed with mean 60 and standard deviation 8. Use $P(0 < Z < 1.28) = 0.40$ and $P(0 < Z < 1.645) = 0.45$, where Z is the standard normal variate.

- (a) The top 10 per cent of the students are to receive a merit certificate. Find the lowest mark that earns a certificate. [2]
- (b) The bottom 5 per cent are to be offered remedial classes. Find the mark below which a student is offered these classes. [2]
- (c) Find the number of students who receive neither a certificate nor remedial classes. [1]

Question 20 [5]

A school in Dehradun plans to replace its bus in 5 years' time, when a new bus is expected to cost Rs 25,00,000. It sets up a sinking fund, depositing an equal sum at the end of every year in an account that pays 8 per cent per annum compounded annually. Use $(1.08)^5 = 1.4693$.

- (a) Find the annual deposit, to the nearest rupee. [3]
- (b) Find the total interest the fund earns. [1]
- (c) State one way in which a sinking fund differs from an ordinary savings account. [1]

OR

Question 20 [5]

In April 2022 Mr Iyer invested Rs 50,000 in a mutual fund. In April 2025 his investment was worth Rs 66,550. His sister invested the same sum at the same time in a fixed deposit paying 9 per cent per annum compounded annually.

- (a) Find the compound annual growth rate (CAGR) of Mr Iyer's investment. [2]
- (b) Find the absolute return on his investment over the three years, as a percentage. [1]
- (c) Find the value of his sister's fixed deposit in April 2025, and state which investment did better and by how much. [2]

ANSWER KEY AND MARKING SCHEME

A full model answer is given for every part. Marks shown in *italics* are the step marks an examiner would award, and they add up to the marks for the part. Method marks are given for a correct method even when an arithmetical slip spoils the final figure, but the final answer mark needs the correct value with its unit (rupees, units, square metres and so on). Accept any correct alternative method that reaches the same answer. In financial questions accept answers that differ only in the last rupee or paise through rounding, and accept values worked with the exact powers instead of the rounded ones printed in the question. For a question with an OR alternative, mark only the alternative the candidate has attempted; if both are attempted, mark both and keep the better mark. Every numerical result in this key has been recomputed by the Future Topper subject team in Python.

Question 1 [20]

- (i) (b) an equivalence relation. Every element is related to itself, (1, 2) and (2, 1) both belong to R, and the only chains (1, 2), (2, 1) and (2, 1), (1, 2) give (1, 1) and (2, 2), which are in R, so R is also transitive.
- (ii) (c) 25. For a square matrix of order n, the determinant of $\text{adj } A$ is the determinant of A raised to the power $n - 1$, here $5^2 = 25$.
- (iii) (a) skew symmetric, since $(A - A')' = A' - A = -(A - A')$.
- (iv) (c) $xe^x(x + 2)$. By the product rule $dy/dx = 2xe^x + x^2e^x = xe^x(x + 2)$.
- (v) (d) $\log|\log x| + c$. Put $t = \log x$, so $dt = dx/x$ and the integral becomes the integral of dt/t .
- (vi) (b) 2.4. The variance is $npq = 10 \times 0.4 \times 0.6 = 2.4$.
- (vii) (c) 12. The values of Z at the corners are 0, 8, 12 and 9, and the largest is 12 at (3, 2).
- (viii) (a) Both are true and the Reason explains the Assertion: $600/0.08 = 7,500$.
- (ix) (a) Rs 45. $C'(x) = 4x + 5$, so $C'(10) = 45$. (Rs 350 is the total cost, not the marginal cost.)
- (x) (a) Both statements are true. $P(A \text{ and not } B) = P(A) - P(A)P(B) = P(A)P(\text{not } B)$, and Statement II is the definition of conditional probability.
- (xi) (b) Rs 9,000. Annual depreciation = $(80,000 - 8,000)/8 = 9,000$.
- (xii) (d) Statement I is false and Statement II is true. $f'(x) = 2x - 4$ is negative on (0, 2), so f decreases there and is not increasing on the whole of (0, 4).
- (xiii) (c) no solution. The determinant of the coefficients is $12 - 12 = 0$, and doubling the first equation gives $4x + 6y = 10$, which contradicts $4x + 6y = 12$, so the system is inconsistent.
- (xiv) (a) e^{-2} . $P(X = 0) = e^{-m} m^0/0! = e^{-2}$.
- (xv) (d) The Assertion is false and the Reason is true. Because the flat rate charges interest on the whole principal throughout, it charges more interest, not less, than the reducing balance method.
- (xvi) (d) not defined, because the equation cannot be written as a polynomial in the derivatives (the second derivative appears inside a logarithm).
- (xvii) (b) $4/3$. Since $k + 2k + 3k = 1$, $k = 1/6$, and the mean is $0 \times 1/6 + 1 \times 2/6 + 2 \times 3/6 = 8/6 = 4/3$.
- (xviii) The price relative is $(52/40) \times 100 = 130$.
- (xix) $2x + 3y \leq 40$ (with $x \geq 0$ and $y \geq 0$).
- (xx) Rate of return = $(270 - 250 + 10)/250 \times 100 = 30/250 \times 100 = 12$ per cent.

1 mark for each correct part. No mark is deducted for a wrong option. In parts (xviii) to (xx) the mark is given for the correct value; a one-line reason is welcome but not required.

Question 2 [2]

$(g \circ f)(x) = g(f(x)) = (3x - 2)^2 + 1 = 9x^2 - 12x + 4 + 1 = 9x^2 - 12x + 5$. For $(f \circ g)(2)$, first $g(2) = 4 + 1 = 5$, then $f(5) = 15 - 2 = 13$. So $(f \circ g)(2) = 13$.
 $(g \circ f)(x) = 9x^2 - 12x + 5$ 1. $(f \circ g)(2) = 13$ 1.

Question 3 [2]

The area is one half of the absolute value of the determinant whose rows are 1, 2, 1; 4, 6, 1; and 7, 3, 1.

Expanding along the first row, the determinant is $1(6 - 3) - 2(4 - 7) + 1(12 - 42) = 3 + 6 - 30 = -21$. Hence the area is $(1/2) \times |-21| = 21/2 = 10.5$ square metres.

Determinant set up and evaluated as -21 (or 21) 1. Area 10.5 square metres, taking the absolute value 1.

OR

Question 3 [2]

The points are collinear when the determinant whose rows are 2, 3, 1; 5, k, 1; and 8, 9, 1 is zero. Expanding along the first row: $2(k - 9) - 3(5 - 8) + 1(45 - 8k) = 2k - 18 + 9 + 45 - 8k = 36 - 6k$. Setting $36 - 6k = 0$ gives $k = 6$. (Check: the slope from (2, 3) to (5, 6) is 1 and from (5, 6) to (8, 9) is 1.)

Determinant equated to zero and expanded to $36 - 6k$ 1. $k = 6$ 1.

Question 4 [2]

$dx/dt = 2$ and $dy/dt = 3t^2$, so $dy/dx = 3t^2/2$. Then $d^2y/dx^2 = [d/dt(3t^2/2)] \times dt/dx = 3t \times (1/2) = 3t/2$. At $t = 2$, $dy/dx = 12/2 = 6$ and $d^2y/dx^2 = 6/2 = 3$.

Note for the examiner: A common error is to stop at $d/dt(dy/dx) = 3t$, giving 6; that loses the second mark because it omits the factor $dt/dx = 1/2$.

$dy/dx = 3t^2/2$, value 6 at $t = 2$ 1. $d^2y/dx^2 = 3t/2$, value 3 at $t = 2$ 1.

Question 5 [2]

Let X be the number of defective bulbs among the 4. Then X is binomial with $n = 4$, $p = 0.1$ and $q = 0.9$. $P(X \leq 1) = P(X = 0) + P(X = 1) = (0.9)^4 + 4 \times (0.1) \times (0.9)^3 = 0.6561 + 4 \times 0.1 \times 0.729 = 0.6561 + 0.2916 = 0.9477$.

Binomial model with $n = 4$, $p = 0.1$ and $P(X = 0) + P(X = 1)$ written down 1. Probability 0.9477 1.

OR

Question 5 [2]

Let X be the number of sixes. X is binomial with $n = 3$, $p = 1/6$ and $q = 5/6$. $P(X = 2) = {}^3C_2 \times (1/6)^2 \times (5/6) = 3 \times (1/36) \times (5/6) = 15/216 = 5/72$.

Binomial set-up ${}^3C_2(1/6)^2(5/6)$ 1. Probability 5/72 1.

Question 6 [2]

(a) This is an ordinary perpetuity, so the present value is $R/i = 30,000/0.06 = \text{Rs } 5,00,000$.

Rs 5,00,000 1.

(b) The trust needs the first payment now as well as an ordinary perpetuity for the later payments, so the sum is $30,000 + 5,00,000 = \text{Rs } 5,30,000$.

Rs 5,30,000 1.

Question 7 [2]

The sum of the base year prices is $30 + 50 + 80 + 40 = 200$ and the sum of the current year prices is $36 + 60 + 105 + 49 = 250$. The simple aggregative price index is $(250/200) \times 100 = 125$. Prices have risen by 25 per cent on this measure.

Both totals, 200 and 250 1. Index 125 1.

Question 8 [2]

For $0 \leq x \leq 2$, $|x - 2| = 2 - x$, and for $2 \leq x \leq 4$, $|x - 2| = x - 2$. So the integral is the integral of $(2 - x)$ from 0 to 2 plus the integral of $(x - 2)$ from 2 to 4 = $[2x - x^2/2]$ from 0 to 2 + $[x^2/2 - 2x]$ from 2 to 4 = $(4 - 2) + [(8 - 8) - (2 - 4)] = 2 + 2 = 4$.

Splitting the range at $x = 2$ with the correct sign on each piece 1. Value 4 1.

OR

Question 8 [2]

Replacing x by $1 - x$, the integral equals the integral of $(1 - x)x^5$ from 0 to 1, that is the integral of $(x^5 - x^6)$ from 0 to 1 = $[x^6/6 - x^7/7]$ from 0 to 1 = $1/6 - 1/7 = 1/42$.

Property applied to get the integral of $x^5 - x^6$ 1. Value 1/42 1.

Question 9 [3]

Taking the tangent of both sides and using the addition formula, $(2x + 3x)/(1 - 6x^2) = \tan(\pi/4) = 1$, provided $6x^2 < 1$. So $5x = 1 - 6x^2$, that is $6x^2 + 5x - 1 = 0$, which factorises as $(6x - 1)(x + 1) = 0$, giving $x = 1/6$ or $x = -1$. For $x = -1$ both terms are negative, so their sum is negative and cannot equal $\pi/4$; also $6x^2 = 6$ is not less than 1. Hence $x = -1$ is rejected and the solution is $x = 1/6$. Check: $\tan^{-1}(1/3) + \tan^{-1}(1/2)$ has tangent $(5/6)/(5/6) = 1$.

Addition formula giving $5x/(1 - 6x^2) = 1$ 1. Quadratic $6x^2 + 5x - 1 = 0$ solved 1. $x = -1$ rejected with a reason; answer $x = 1/6$ 1.

OR

Question 9 [3]

First, $2 \tan^{-1}(1/2) = \tan^{-1}[2(1/2)/(1 - 1/4)] = \tan^{-1}[1/(3/4)] = \tan^{-1}(4/3)$, valid since $1/2$ lies between -1 and 1 .

Then $\tan^{-1}(4/3) - \tan^{-1}(1/7) = \tan^{-1}[(4/3 - 1/7)/(1 + (4/3)(1/7))] = \tan^{-1}[(25/21)/(25/21)] = \tan^{-1}(1) = \pi/4$. Hence the result.

$2 \tan^{-1}(1/2) = \tan^{-1}(4/3)$ 1. Subtraction formula applied correctly 1. Value $\tan^{-1}(1) = \pi/4$ 1.

Question 10 [3]

Revenue $R(x) = px = 80x - 0.02x^2$, so profit $P(x) = R(x) - C(x) = 60x - 0.02x^2 - 5000$. Then $P'(x) = 60 - 0.04x$, which is zero when $x = 1500$, and $P''(x) = -0.04 < 0$, so $x = 1500$ gives a maximum. The price is $p = 80 - 0.02 \times 1500 = \text{Rs } 50$ per litre, and the greatest profit is $P(1500) = 90,000 - 45,000 - 5,000 = \text{Rs } 40,000$ a day.

Profit function $60x - 0.02x^2 - 5000$ 1. $x = 1500$ litres with the second derivative test 1. Price Rs 50 and profit Rs 40,000 1.

Question 11 [3]

Write $(5x + 4)/((x - 1)(x + 2)) = A/(x - 1) + B/(x + 2)$, so $5x + 4 = A(x + 2) + B(x - 1)$. Putting $x = 1$ gives $9 = 3A$, so $A = 3$; putting $x = -2$ gives $-6 = -3B$, so $B = 2$. Hence the integral is the integral of $[3/(x - 1) + 2/(x + 2)] dx = 3 \log|x - 1| + 2 \log|x + 2| + c$.

Correct partial fraction form 1. $A = 3$ and $B = 2$ 1. $3 \log|x - 1| + 2 \log|x + 2| + c$ 1.

OR

Question 11 [3]

Integrate by parts with x as the first function: the indefinite integral is $x(e^{2x}/2)$ minus the integral of $e^{2x}/2 dx = (x/2)e^{2x} - e^{2x}/4$. Between the limits: at $x = 1$ this is $e^2/2 - e^2/4 = e^2/4$, and at $x = 0$ it is $0 - 1/4 = -1/4$. So the value is $e^2/4 + 1/4 = (e^2 + 1)/4$, which is about 2.097.

Integration by parts set up correctly 1. Antiderivative $(x/2)e^{2x} - e^{2x}/4$ 1. Value $(e^2 + 1)/4$ 1.

Question 12 [3]

The number of ways to choose 2 chargers from 6 is ${}^6C_2 = 15$. $P(X = 0) = {}^4C_2/15 = 6/15$, $P(X = 1) = (4 \times 2)/15 = 8/15$ and $P(X = 2) = {}^2C_2/15 = 1/15$; these add to 1. Mean $= 0 \times 6/15 + 1 \times 8/15 + 2 \times 1/15 = 10/15 = 2/3$. $E(X^2) = 0 + 8/15 + 4/15 = 12/15 = 4/5$. Variance $= 4/5 - (2/3)^2 = 4/5 - 4/9 = 16/45$, which is about 0.356.

Distribution 6/15, 8/15, 1/15 1. Mean 2/3 1. Variance 16/45 1.

Question 13 [3]

- (a) $EMI = P \times i \times (1 + i)^n / [(1 + i)^n - 1]$ with $P = 6,00,000$, $i = 0.01$ and $n = 36$. So $EMI = 6,00,000 \times 0.01 \times 1.4308/0.4308 = 8,584.80/0.4308 = \text{Rs } 19,927.58$, that is about Rs 19,928.

Correct EMI formula with $i = 0.01$ and $n = 36$ 1. EMI about Rs 19,928 1.

- (b) Total paid $= 36 \times 19,927.58 = \text{Rs } 7,17,392.88$, so the total interest $= 7,17,392.88 - 6,00,000 = \text{Rs } 1,17,392.88$, about Rs 1,17,393 (accept Rs 1,17,408 if the EMI was rounded to Rs 19,928 first, and Rs 1,17,429 if the exact power was used, giving EMI Rs 19,928.59).

Total interest about Rs 1,17,393 1.

Note for the examiner: With the exact power $(1.01)^{36} = 1.430769$ the EMI is Rs 19,928.59; accept either value.

OR

Question 13 [3]

- (a) Under the flat rate method interest is charged on the full principal for the whole term: interest = $1,20,000 \times 0.09 \times 2 = \text{Rs } 21,600$, and the instalment = $(1,20,000 + 21,600)/24 = 1,41,600/24 = \text{Rs } 5,900$.

Interest Rs 21,600 and instalment Rs 5,900 1.

- (b) $\text{EMI} = P \times i \times (1 + i)^n / [(1 + i)^n - 1] = 1,20,000 \times 0.0125 \times 1.3474 / 0.3474 = 2,021.10 / 0.3474 = \text{Rs } 5,817.79$, about Rs 5,818.

EMI about Rs 5,818 1.

- (c) Total interest with Q = $24 \times 5,817.79 - 1,20,000 = 1,39,626.96 - 1,20,000 = \text{Rs } 19,626.96$. Lender Q is cheaper by $21,600 - 19,626.96 = \text{Rs } 1,973.04$, because the flat rate charges interest on the full Rs 1,20,000 throughout while the reducing balance rate charges only on what is still owed.

Interest about Rs 19,627 with Q named cheaper by about Rs 1,973 1.

Note for the examiner: With the exact power $(1.0125)^{24} = 1.347351$ the EMI is Rs 5,818.40, the interest Rs 19,641.55 and the saving Rs 1,958.45; accept either set of figures.

Question 14 [3]

Draw $x + y = 8$ through (8, 0) and (0, 8), $x + 2y = 12$ through (12, 0) and (0, 6), and the vertical line $x = 6$. The feasible region is the part of the first quadrant on the origin side of all three lines. Its corners are (0, 0), (6, 0), the meeting point of $x = 6$ and $x + y = 8$, which is (6, 2), the meeting point of $x + y = 8$ and $x + 2y = 12$, which is (4, 4) (subtracting gives $y = 4$), and (0, 6). Values of Z: $Z(0, 0) = 0$, $Z(6, 0) = 30$, $Z(6, 2) = 38$, $Z(4, 4) = 36$, $Z(0, 6) = 24$. The region is bounded, so the maximum is 38, at $x = 6$ and $y = 2$.

Feasible region drawn and shaded correctly 1. All five corner points, including (6, 2) and (4, 4) 1. Maximum Z = 38 at (6, 2) 1.

Question 15 [3]

The three-yearly moving totals, placed against the middle year, are 2019: $42 + 48 + 45 = 135$; 2020: $48 + 45 + 51 = 144$; 2021: $45 + 51 + 57 = 153$; 2022: $51 + 57 + 54 = 162$; 2023: $57 + 54 + 60 = 171$. Dividing by 3, the moving averages are 45, 48, 51, 54 and 57 thousand tonnes for 2019 to 2023. On the graph the original figures zig-zag, while the moving averages lie on a straight line rising by 3 thousand tonnes a year, showing a steady upward trend.

Moving totals 135, 144, 153, 162, 171 placed against the middle years 1. Moving averages 45, 48, 51, 54, 57 1. Both series plotted and the rising trend stated 1.

Question 16 [5]

- (a) The equations are $x + y + z = 190$, $2x + y + 3z = 450$ and $x + 2y + z = 240$. So A is the matrix with rows 1, 1, 2; 2, 1, 3; 1, 2, 1, X is the column x, y, z and B is the column 190, 450, 240. Expanding along the first row, the determinant of $A = 1(1 - 6) - 1(2 - 3) + 1(4 - 1) = -5 + 1 + 3 = -1$, which is not zero, so A^{-1} exists. The cofactors are $A_{11} = -5$, $A_{12} = 1$, $A_{13} = 3$, $A_{21} = 1$, $A_{22} = 0$, $A_{23} = -1$, $A_{31} = 2$, $A_{32} = -1$, $A_{33} = -1$. The adjoint (transpose of the cofactor matrix) has rows -5, 1, 2; 1, 0, -1; 3, -1, -1. Hence $A^{-1} = (1/-1) \text{ adj } A$, the matrix with rows 5, -1, -2; -1, 0, 1; -3, 1, 1.

Matrix equation and determinant -1 1. Cofactors and adjoint 1. A^{-1} with rows 5, -1, -2; -1, 0, 1; -3, 1, 1 1.

- (b) $X = A^{-1}B$: $x = 5 \times 190 - 450 - 2 \times 240 = 950 - 450 - 480 = 20$; $y = -190 + 0 + 240 = 50$; $z = -3 \times 190 + 450 + 240 = -570 + 690 = 120$. A pen costs Rs 20, a notebook Rs 50 and a geometry box Rs 120. (Check in Kabir's bill: $40 + 50 + 360 = 450$.)

$x = 20$, $y = 50$, $z = 120$ 1.

- (c) Cost = $5 \times 20 + 4 \times 50 = 100 + 200 = \text{Rs } 300$.

Rs 300 1.

Question 17 [5]

- (a) It is of the form $dy/dx + Py = Q$ with $P = 2/x$ and $Q = 3x$. The integrating factor is e to the power of the integral of $(2/x) dx$, that is $e^{2 \log x} = x^2$.

$P = 2/x$, $Q = 3x$ and integrating factor x^2 1.

- (b) Multiplying through by x^2 , $x^2(dy/dx) + 2xy = 3x^3$, that is $d/dx(x^2y) = 3x^3$. Integrating, $x^2y = 3x^4/4 + c$, so $y = 3x^2/4 + c/x^2$.

Left side recognised as $d/dx(x^2y)$ 1. Integration giving $x^2y = 3x^4/4 + c$ 1. General solution $y = 3x^2/4 + c/x^2$ 1.

- (c) Putting $x = 1$ and $y = 2$: $2 = 3/4 + c$, so $c = 5/4$. The particular solution is $y = 3x^2/4 + 5/(4x^2)$. Check: $dy/dx = 3x/2 - 5/(2x^3)$, and adding $(2/x)y = 3x/2 + 5/(2x^3)$ gives $3x$.

$c = 5/4$ and the particular solution 1.

Question 18 [5]

- (a) At equilibrium $50 - x^2 = 2x + 2$, so $x^2 + 2x - 48 = 0$, that is $(x + 8)(x - 6) = 0$. Rejecting the negative root, $x_0 = 6$ and $p_0 = 2 \times 6 + 2 = 14$.

$x_0 = 6$ and $p_0 = 14$ 1.

- (b) Consumer surplus = integral of $(50 - x^2)$ from 0 to 6 minus $p_0x_0 = [50x - x^3/3]$ from 0 to 6 – $14 \times 6 = (300 - 72) - 84 = 228 - 84 = 144$.

Correct formula with limits 0 to 6 1. Consumer surplus 144 1.

- (c) Producer surplus = p_0x_0 minus the integral of $(2x + 2)$ from 0 to 6 = $84 - [x^2 + 2x]$ from 0 to 6 = $84 - (36 + 12) = 84 - 48 = 36$.

Correct formula with limits 0 to 6 1. Producer surplus 36 1.

Note for the examiner: The units are hundreds of rupees times thousands of units, so 144 corresponds to Rs 1,44,00,000 (one crore forty-four lakh rupees) and 36 to Rs 36,00,000. Do not penalise a candidate who leaves the answers as 144 and 36 in the stated units.

OR

Question 18 [5]

- (a) $C(x)$ is the integral of $(4x + 20) dx = 2x^2 + 20x + k$. The fixed cost is the cost at $x = 0$, so $k = 600$, giving $C(x) = 2x^2 + 20x + 600$.

Marginal cost integrated with constant 600, $C(x) = 2x^2 + 20x + 600$ 1.

- (b) Revenue $R(x) = 100x$. At break-even $R(x) = C(x)$: $100x = 2x^2 + 20x + 600$, so $2x^2 - 80x + 600 = 0$, that is $x^2 - 40x + 300 = 0$, which gives $(x - 10)(x - 30) = 0$. The break-even points are $x = 10$ and $x = 30$ bottles a day.

Equation $R(x) = C(x)$ reduced to $x^2 - 40x + 300 = 0$ 1. $x = 10$ and $x = 30$ 1.

- (c) Profit $P(x) = 80x - 2x^2 - 600$, so $P'(x) = 80 - 4x = 0$ at $x = 20$, with $P''(x) = -4 < 0$. The greatest profit is $P(20) = 1600 - 800 - 600 = \text{Rs } 200$ a day.

$x = 20$ with the second derivative test 1. Greatest profit Rs 200 1.

Question 19 [5]

- (a) Let S, R and E be the events that the parcel goes by Swift, Rapid and Express, and L the event that it is late. By the theorem of total probability, $P(L) = P(S)P(L \text{ given } S) + P(R)P(L \text{ given } R) + P(E)P(L \text{ given } E) = 0.5 \times 0.02 + 0.3 \times 0.05 + 0.2 \times 0.10 = 0.010 + 0.015 + 0.020 = 0.045$.

Total probability set up 1. $P(L) = 0.045$ 1.

- (b) By Bayes' theorem, $P(E \text{ given } L) = P(E)P(L \text{ given } E)/P(L) = 0.020/0.045 = 4/9$, which is about 0.444.

Bayes' theorem applied 1. $4/9$ 1.

- (c) $P(\text{not } L) = 1 - 0.045 = 0.955$ and $P(S \text{ and not } L) = 0.5 \times 0.98 = 0.49$. So $P(S \text{ given not } L) = 0.49/0.955 = 490/955 = 98/191$, which is about 0.513.

$98/191$ (about 0.513) 1.

OR

Question 19 [5]

- (a) We need the mark k with $P(X > k) = 0.10$. Since $P(Z > 1.28) = 0.5 - 0.40 = 0.10$, $(k - 60)/8 = 1.28$, so $k = 60 + 1.28 \times 8 = 60 + 10.24 = 70.24$. A student needs about 70.24 marks, that is 71 marks if marks are whole numbers.

$P(Z > 1.28) = 0.10$ used to set $(k - 60)/8 = 1.28$ 1. Cut-off 70.24 marks 1.

- (b) We need m with $P(X < m) = 0.05$. Since $P(Z < -1.645) = 0.5 - 0.45 = 0.05$, $(m - 60)/8 = -1.645$, so $m = 60 - 13.16 = 46.84$ marks.

Negative Z value -1.645 used 1. Mark 46.84 1.

- (c) The proportion between the two cut-offs is $1 - 0.10 - 0.05 = 0.85$, so the number is $0.85 \times 500 = 425$ students.

425 students 1.

Note for the examiner: If a candidate rounds the cut-off in (a) up to 71 marks for whole-number marks, accept it.

Question 20 [5]

- (a) The amount of an annuity of R per year for n years at rate i is $R[(1 + i)^n - 1]/i$. Here $25,00,000 = R(1.4693 - 1)/0.08 = R \times 0.4693/0.08 = R \times 5.86625$. So $R = 25,00,000 \times 0.08/0.4693 = 2,00,000/0.4693 = \text{Rs } 4,26,166.63$, that is about Rs 4,26,167.

Annuity amount formula quoted 1. Correct substitution with $0.4693/0.08$ 1. Annual deposit about Rs 4,26,167 1.

- (b) Total deposited = $5 \times 4,26,167 = \text{Rs } 21,30,835$, so the interest earned = $25,00,000 - 21,30,835 = \text{Rs } 3,69,165$. (With the exact power the deposit is Rs 4,26,141 and the interest Rs 3,69,295; accept either.)

Interest about Rs 3,69,165 1.

- (c) A sinking fund is set up for a specific known future liability, with fixed periodic deposits calculated to reach a target sum by a fixed date, whereas a savings account has no fixed target, date or deposit amount and money may be withdrawn at any time.

Valid distinction 1.

Note for the examiner: With the exact power $(1.08)^5 = 1.469328$ the deposit is Rs 4,26,141; accept either.

OR

Question 20 [5]

- (a) $\text{CAGR} = (\text{final value}/\text{initial value})^{1/n} - 1$ with $n = 3$. $66,550/50,000 = 1.331$, and since $1.1^3 = 1.331$, the cube root is 1.1. So $\text{CAGR} = 1.1 - 1 = 0.10$, that is 10 per cent per annum.

CAGR formula with $n = 3$ and ratio 1.331 1. CAGR 10 per cent 1.

- (b) Absolute return = $(66,550 - 50,000)/50,000 \times 100 = 16,550/50,000 \times 100 = 33.1$ per cent.

33.1 per cent 1.

- (c) Value = $50,000 \times (1.09)^3 = 50,000 \times 1.295029 = \text{Rs } 64,751.45$. The mutual fund did better, by $66,550 - 64,751.45 = \text{Rs } 1,798.55$, which matches its higher growth rate of 10 per cent against 9 per cent.

Fixed deposit value Rs 64,751.45 1. Mutual fund better by Rs 1,798.55 1.